

Reliability Analysis of On-orbit Life of Satellite-borne Liquid Floated Gyroscope with Superhydrophobic Surface Material

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Abstract: In the past few years, the superhydrophobic surface material has been widely used in the satellite-borne liquid floated gyroscope, but the actual reliability of on-orbit life of satellite-borne liquid floated gyroscope on this type is insufficient. This paper addresses the problem of insufficient research on the actual on-orbit life reliability of a certain type of liquid floated gyroscope in China. Based on the actual on-orbit life data of the gyroscope, reliability analysis model was established using Weibull distribution, parameter estimation was made by combining mean rank order to realize life reliability analysis and obtain fault rate and reliability under different years. The results show that: In China, a certain type of superhydrophobic surface material liquid floated gyroscope, under the current allocation quantity of satellite gyroscope, it will be impossible for this type of satellite-borne gyroscope to meet the normal use requirements when it is in orbit for 5.5 years. Before this time limit, other alternative attitude determination means need to be considered. The minimum attitude determination requirement will not be met in the case of on-orbit service for 7.5 years. The satellite mode of operation will be affected when there is no alternative attitude determination means. The analysis results in this paper provide an effective way to correctly evaluate the reliability index of liquid floated gyroscope, which offers a reference for its on-orbit application.

Keywords: liquid floated gyroscope; on-orbit life; reliability

1. Introduction

Superhydrophobic surface research has attracted more and more attention from material scientists, and this technology has also been introduced into the study of gyro surfaces[1-4]. Satellite-borne gyroscope constitutes the core component of the satellite inertial attitude sensor. It outputs the angular rate of the satellite's three axes relative to the inertial space, and performs satellite attitude determination in conjunction with other sensors such as star sensors. There are mechanical gyroscope, optical gyroscope, micro-mechanical gyroscope and so on. Liquid floated gyroscope(LFG) represents a typical mechanical gyroscope which is widely used in different types of

satellites at home and abroad. LFG plays a very important role in the inertial navigation system [5]. A certain type of domestic liquid floated gyroscope with superhydrophobic surface material currently used in satellites in China has a design life of less than 3 years, while the one with the longest has been working in orbit for more than 8 years. Big individual difference exists in actual on-orbit life of this type of gyroscope, and no obvious law is shown in distribution. No clear conclusion has been reached in its actual on-orbit life reliability study.

In this paper, with the actual on-orbit life data of a certain type of liquid floated gyroscope with superhydrophobic surface material as a sample, reliability analysis model was established using Weibull distribution, parameter estimation was made using least squares method in conjunction with mean rank order to analyze and predict the life reliability. The analysis results in this paper provide an effective way to correctly evaluate the reliability index of liquid floated gyroscope. The results have great value in the aspect of On-orbit application of this type of liquid floated gyroscope. On the one hand, it provides a basis for predicting the life reliability of the on-orbit gyroscope, and on the other hand, it provides a reference for the application effect of the superhydrophobic surface material in the liquid floating gyroscope.

2. Data sources

A certain type of liquid floated gyroscopes with over 5 years' on-orbit life under the same service life and working conditions were selected as the analysis samples. The gyroscope life data was derived from the power-on working time, that is, the duration from the initial power-on time after orbit insertion to the final power-off time. The final power-off means the last power-off of a gyroscope within the observed time range, including two cases of fault power-off and non-fault power-off. The latter is referred to as right censored data in the theory of survival analysis, that is, the exact fault time is unknown but its lifetime is known to exceed its actual power-on working time. In addition, data of gyroscope that does not end its life until the observation deadline can also be regarded as right censored data.

The life data is derived from 48 liquid floated gyroscopes that meet the conditions. The actual working

conditions are shown in Table 1. The Weibull distribution parameter estimation method adopted in this paper requires reordering of the above life data in ascending

order, and the fault data is reflected in the “status” based on the order of its fault occurrence time.

Table 1 On-orbit working time and fault data of a certain type of liquid floated gyroscope

No.	On-orbit working time(year)	Status	No.	On-orbit working time(year)	Status	No.	On-orbit working time(year)	Status	No.	On-orbit working time(year)	Status
1	0.37 ⁺	W ₁	13	2.93 ⁺	W ₄	25	3.89	D ₁₈	37	5.87 ⁺	W ₁₅
2	0.72	D ₁	14	3.14	D ₁₀	26	4.28 ⁺	W ₈	38	5.92 ⁺	W ₁₆
3	1.61 ⁺	W ₂	15	3.18	D ₁₁	27	4.46	D ₁₉	39	5.92 ⁺	W ₁₇
4	1.73	D ₂	16	3.18	D ₁₂	28	4.6	D ₂₀	40	5.93	D ₂₃
5	2.1	D ₃	17	3.22	D ₁₃	29	4.95	D ₂₁	41	6.22	D ₂₄
6	2.27	D ₄	18	3.23 ⁺	W ₅	30	5.04 ⁺	W ₉	42	6.31 ⁺	W ₁₈
7	2.45	D ₅	19	3.28	D ₁₄	31	5.12	D ₂₂	43	6.65 ⁺	W ₁₉
8	2.56 ⁺	W ₃	20	3.34	D ₁₅	32	5.43 ⁺	W ₁₀	44	7.01	D ₂₅
9	2.71	D ₆	21	3.5	D ₁₆	33	5.58 ⁺	W ₁₁	45	7.06	D ₂₆
10	2.85	D ₇	22	3.53 ⁺	W ₆	34	5.60 ⁺	W ₁₂	46	7.56 ⁺	W ₂₀
11	2.85	D ₈	23	3.64	D ₁₇	35	5.65 ⁺	W ₁₃	47	8.55 ⁺	W ₂₁
12	2.87	D ₉	24	3.82 ⁺	W ₇	36	5.69 ⁺	W ₁₄	48	8.82 ⁺	W ₂₂

Note: D indicates fault data, W indicates censored data, and data with the corner mark "+" indicates censored data.

3. Analysis methods

As the gyroscope's on-orbit time gets longer, the fault rate increases and the reliability gradually decreases. The fault rate and reliability are analyzed based on the actual on-orbit life data, which provides an effective way to correctly evaluate the gyroscope reliability index.

Weibull distribution serves as the theoretical basis of reliability analysis and life testing [6], which is especially suitable for the analysis of the cumulative wear fault of mechanical and electrical products. Capable of using probability value to infer its distribution parameters, it is widely used in various types of life data processing. Liquid floated gyroscope is a typical mechanical gyroscope whose fault rate function obeys Weibull distribution.

Weibull fault distribution function:

$$F(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^m} \tag{1}$$

Fault density function:

$$f(t) = \frac{mt^{m-1}}{\eta^m} e^{-\left(\frac{t}{\eta}\right)^m} \tag{2}$$

Reliability function:

$$R(t) = e^{-\left(\frac{t}{\eta}\right)^m} \tag{3}$$

Fault rate function:

$$\lambda(t) = \frac{m}{\eta} \left(\frac{t}{\eta}\right)^{m-1} \tag{4}$$

T is time, η is scale parameter, m is shape parameter [7].

When $m < 1$, fault rate $\lambda(t)$ shows decreasing distribution. When $m = 1$, fault rate $\lambda(t)$ is constant, and when $m > 1$, fault rate $\lambda(t)$ shows increasing distribution. Weibull parameters are usually estimated using moment estimation method, the least square method and the maximum likelihood method[8], etc. In this paper,

Weibull parameters are estimated using least square method in conjunction with mean rank order[9].

Least squares estimation represents a better method for Weibull distribution parameter estimation.

The formula (4) is subject to left and right deformation:

$$e^{\left(\frac{t}{\eta}\right)^m} = \frac{1}{1 - F(t)} \tag{5}$$

Take the natural logarithm on both sides and then:

$$\ln\left(\ln\left(\frac{1}{1 - F(t)}\right)\right) = m(\ln t - \ln \eta) \tag{6}$$

Let

$$x = \ln t \quad y = \ln\left(\ln\left(\frac{1}{1 - F(t)}\right)\right) \quad A = \eta \quad B = -m \ln \eta \tag{7}$$

Then $Y = AX + B$, the least squares estimation solution of the regression coefficients A and B is:

$$\begin{cases} \hat{A} = \frac{\sum_{i=1}^n x_i y_i - n \bar{x} \bar{y}}{\sum_{i=1}^n x_i^2 - n \bar{x}^2} \\ \hat{B} = \bar{y} - \hat{A} \bar{x} \end{cases} \tag{8}$$

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \quad \bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$$

In the formula

The empirical distribution function is usually calculated by the approximate median rank formula

$$F_n(t_i) = \frac{i - 0.3}{n + 0.4}$$

, but the error is big. Moreover, in the case of censored data, the sample data has incomplete life. For some samples without fault or mid-way shutdown, the actual fault time is unknown and approximate median rank formula is inapplicable. In order to make full use of all sample data, calculation can be made using mean rank order. Mean rank order is to estimate all possible ranks and solve mean rank based on fault sample and suspension sample, substitute the mean rank into the approximate

median rank formula to solve the empirical distribution function.

The incremental formula [10] of the sample data mean rank with suspension item is as follows:

$$\begin{cases} \Delta A_i = \frac{n+1-A_{i-1}}{n-k+2} \\ A_i = A_{i-1} + \Delta A_i \end{cases} \quad (9)$$

n is the sample size, k is the sequence number of all equipment, which is arranged according to fault time and censored time. i is the sequence number of the fault equipment, A_i is mean rank of fault equipment, A_{i-1} is mean rank of the previous fault equipment.

$$F_n(t_i) = \frac{A_i - 0.3}{n + 0.4} \quad (10)$$

Table 2 Mean rank order and distribution function solution results

Fault number	Mean rank parameter			$x = \ln t$	$y = \ln(\ln(\frac{1}{1-F(t)}))$
	t_i	A_i	$F_n(t_i)$		
1.	0.72	1	0.014	-0.329	-4.229
2.	1.73	2.043	0.036	0.548	-3.306
3.	2.1	3.086	0.058	0.742	-2.825
4.	2.27	4.13	0.079	0.820	-2.496
5.	2.45	5.173	0.101	0.896	-2.243
6.	2.71	6.242	0.123	0.997	-2.033
7.	2.85	7.31	0.145	1.047	-1.855
8.	2.85	8.38	0.167	1.047	-1.700
9.	2.87	9.45	0.189	1.054	-1.563
10.	3.14	10.55	0.212	1.144	-1.436
11.	3.18	11.65	0.235	1.157	-1.320
12.	3.18	12.75	0.257	1.157	-1.213
13.	3.22	13.85	0.280	1.169	-1.113
14.	3.28	14.98	0.303	1.188	-1.018
15.	3.34	16.11	0.327	1.206	-0.928
16.	3.5	17.24	0.350	1.253	-0.842
17.	3.64	18.42	0.374	1.292	-0.757
18.	3.89	19.64	0.400	1.358	-0.673
19.	4.46	20.92	0.426	1.495	-0.588
20.	4.6	22.2	0.452	1.526	-0.507
21.	4.95	23.48	0.479	1.599	-0.428
22.	5.12	24.82	0.507	1.633	-0.347
23.	5.93	27.24	0.557	1.780	-0.207
24.	6.22	29.66	0.607	1.828	-0.069
25.	7.01	32.88	0.673	1.947	0.112
26.	7.06	36.1	0.740	1.954	0.297

4.1 Model parameter determination

Substitute x and y values obtained in Table 2 into equation (8) and obtain regression coefficient A = 2.192 and B = - 3.937. Then, according to equation (7), the

Weibull distribution parameter is $\eta = e^{-\frac{B}{m}} = 6.026$, $m = A = 2.192$.

4.2 Function determination

The fault time and the calculated empirical distribution function are fitted for regression line of the Weibull distribution model by the least squares parameter estimation method to determine the scale parameters and shape parameters of Weibull distribution.

4. Analysis result

With life data of the 48 gyroscopes in Table 2 as the conceptual data, gyroscope with censored data was removed first, and gyroscope with fault was re-sorted according to the order of fault occurrence time. Weibull distribution model was established, and mean rank and empirical distribution function values of the faulty engine were calculated by the equations (9) and (10) via mean rank order, as shown in the third and fourth columns in Table 2

Weibull fault distribution function:

$$F(t) = 1 - e^{-\left(\frac{t}{6.026}\right)^{2.192}} = 1 - e^{-0.0195t^{2.192}}$$

Fault density function

$$f(t) = \frac{2.192t^{1.192}}{6.026^{2.192}} e^{-\left(\frac{t}{6.026}\right)^{2.192}} = 0.0428t^{1.192} e^{-0.0195t^{2.192}}$$

Reliability function: $R(t) = e^{-\left(\frac{t}{6.026}\right)^{2.192}} = e^{-0.0195t^{2.192}}$

$$\lambda(t) = \frac{2.192}{6.026} \left(\frac{t}{6.026} \right)^{1.192} = 0.0428t^{1.192}$$

4.3 Calculation results

According to the above function, the fault rate and reliability corresponding to different working years of the

gyroscope are calculated, as shown in Table 3. The life data is up to 8.82 years. Starting from the ninth year, and the calculation result is the prediction result. From the perspective of redundancy, the whole satellite is usually equipped with six gyroscopes of this type and at least four gyroscope start-ups are required at the same time. Combining the fault rate, the number of faultless gyroscopes corresponding to different years can be known.

Table 3 Fault rates and reliability for different years

Working time (year)	Fault rate	Reliability	Number of faultless gyroscopes	Working time (year)	Fault rate	Reliability	Number of faultless gyroscopes
0.5	0.018733	0.995742	5.887602	5.5	0.326555	0.441183	4.04067
1	0.0428	0.980689	5.7532	6	0.362243	0.371486	3.826542
1.5	0.069398	0.95368	5.583612	6.5	0.398507	0.30723	3.608958
2	0.097785	0.914751	5.41329	7	0.435312	0.249494	3.388128
2.5	0.127582	0.864747	5.234508	7.5	0.472625	0.198893	3.16425
3	0.158552	0.805161	5.048688	8	0.510419	0.155609	2.937486
3.5	0.190534	0.737986	4.856796	8.5	0.54867	0.119455	2.70798
4	0.223408	0.665547	4.659552	9	0.587355	0.089956	2.47587
4.5	0.257083	0.590326	4.577502	9.5	0.626456	0.066439	2.241264
5	0.291485	0.514781	4.25109	10	0.665953	0.048116	2.004282

5. Discussion and conclusion

Fault rate and reliability of a certain type of liquid floated gyroscope with superhydrophobic surface material can be well estimated and predicted using Weibull distribution parameter estimation method in conjunction with mean rank order formula under the existing in-orbit life data samples. Under the premise of configuring 6 gyroscopes of this type and requiring start-up of a minimum of 4 gyroscopes, the conclusion is: the number of faultless gyroscopes is 4 after on-orbit life for 5.5 years, which will not meet the requirements of normal use soon, and alternative attitude determination means should be considered before this time limit; there are only 3 faultless gyroscopes after on-orbit life for 7.5 years, and the fault rate has exceeded 50%. That is, the whole satellite has lost half of the gyroscope, and the minimum attitude determination requirements cannot be met. The satellite mode of operation will be affected if there is no alternative attitude determination means.

The life data is up to 8.82 years. Subsequently, as the gyroscope's overall on-orbit time increases, the observation time can be extended, and reliability of the prediction can be tested for accuracy by using non-parametric method [11] of survival analysis theory in combination with new fault data.

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